Unit 4: Geometry

Overview (Big Ideas):
Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angle in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres. (CCSS Grade 8 page 53)

Enduring Understandings:
1. Decomposing and rearranging provide a geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one.
2. In addition to decomposing and rearranging, shearing provides another geometric way of both seeing that a measurement formula is the right one and seeing why it is the right one.
3. Geometric images provide the content in relation to which properties can be noticed, definitions can be made, and invariances can be discerned.
4. Symmetry provides a powerful way of working geometrically.
5. Geometric awareness develops through practice in visualizing, diagramming, and constructing.
6. Tools provide new sources of imagery as well as specific ways of thinking about geometric objects and processes.
7. Geometric thinking turns tools into objects, and in geometry the process of turning an action undertaken with a tool into an object happens over and over again.
8. Naming is not just about nomenclature; it draws attention to properties and objects of geometric interest.
9. Definition can both generate and reflect structure: definitions are often dependent on a specific classification.
10. Conjectures can emerge out of a problem-posing process that generates claims that need to be justified.

Source:

Essential Questions:
- What changes do line segments in the coordinate plane undergo that have been rotated, reflected, or translated?
- How do similar figures compare?
What transformations can figures undergo that enable them to remain congruent to the original image?

What are the differences between rigid and non-rigid transformations?

What special angles are formed from parallel lines cut by a transversal?

What characteristics exist among all triangles?

What is the relationship between the lengths of the segments of right triangles?

Curriculum Standards:

**Understand congruence and similarity using physical models, transparencies, or geometry software.**
1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

**Understand and apply the Pythagorean Theorem.**
6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

**Common Misconceptions:**
- Students confuse the rules for transforming two-dimensional figures because they rely too heavily on rules as opposed to understanding what happens to figures as they translate, rotate, reflect, and dilate. It is important to have students describe the effects of
each of the transformations on two-dimensional figures through the coordinates but also the visual transformations that result.

- By definition, congruent figures are also similar. It is incorrect to say that similar figures are the same shape, just a different size. This thinking leads students to misconceptions such as that all triangles are similar. It is important to add to that definition, the property of proportionality among similar figures.

- Student errors with the Pythagorean theorem may involve mistaking one of the legs as the hypotenuse, multiplying the legs and the hypotenuse by 2 as opposed to squaring them, or using the theorem to find missing sides for a triangle that is not right.